PARAMETRIC DICTIONARY LEARNING IN DIFFUSION MRI

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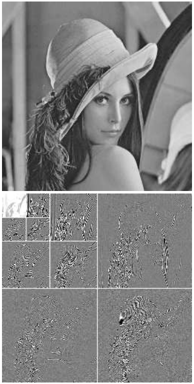
What is sparsity useful for?



A signal has a *sparse* representation if

- it is described with few coefficients,
- the representation is well adapted.

Sparse representations are useful for:

- Data compression (*e.g.* JPEG2000),

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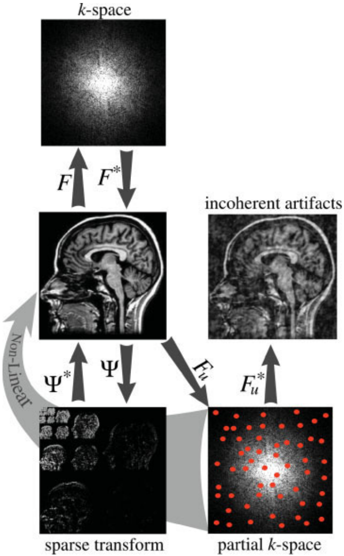
Sparse representations are useful for:

- Data compression (*e.g.* JPEG2000),
- Denoising,
- Compressive sensing.



M Lustig (MRM07)

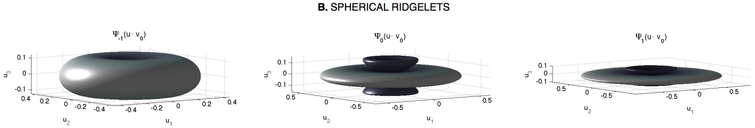
Some sparse transforms in diffusion MRI

Sparse reconstruction problem

$$\arg\min_{\mathbf{x}} ||\mathbf{y} - \mathbf{\Psi x}|| + \lambda ||\mathbf{x}||_1. \tag{1}$$

Continuous transforms

- Spherical wavelets/ridgelets [I Kezele (MedIA 2010), O Michailovitch (ITIP 2009)]
- under-sampled DSI with wavelets [M Menzel (MRM 2011)]



Can we get higher sparsity rate than with these on-the-shelf dictionaries?

Parametric Dictionary Learning - I

Parametric estimation with sparsifying transform

$$||\mathbf{y} - \mathbf{HDx}||_2^2 + \lambda ||\mathbf{x}||_1.$$

- $\mathbf{q}_n, n = 1 \dots N$ is the acquisition sequence
- $\mathbf{H}$ is the observation matrix, $H_{n,j} = f_j(\mathbf{q}_n)$,
- $f_j, j = 1 \dots R$ is the basis of functions,
- $\mathbf{D}$ is a $R \times M$ sparsifying transform.
- λ is calculated per-voxel, using K-fold cross-validation.

Parametric Dictionary Learning - II

Find a dictionary $\mathbf{D}$ of M atoms, given a training data $\mathbf{Y}$

- diffusion signal from S different voxels, $y_{n,s} = E_s(\mathbf{q}_n)$
- $\mathbf{q}_n, k = 1 \dots N$ is the acquisition sequence

First estimate the coefficients $\mathbf{C}$

$$\mathbf{C} = \arg\min_{\mathbf{C}} ||\mathbf{Y} - \mathbf{HC}||_2^2$$

Joint sparse transform and coding problem (solved using K-SVD)

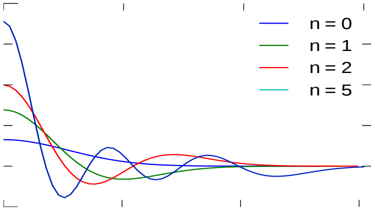
$$\arg\min_{\mathbf{D},\mathbf{X}} ||\mathbf{C} - \mathbf{DX}||_2^2 + \alpha||\mathbf{X}||_1, \ \text{s.t.}\ \forall m \leq M, ||\mathbf{d}_m||_2 = 1$$

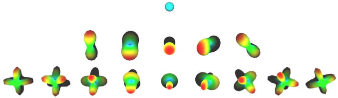
Representation with familiar functions

Spherical Polar Fourier basis [HE Assemlal (MedIA 2009)] to construct dictionary atoms

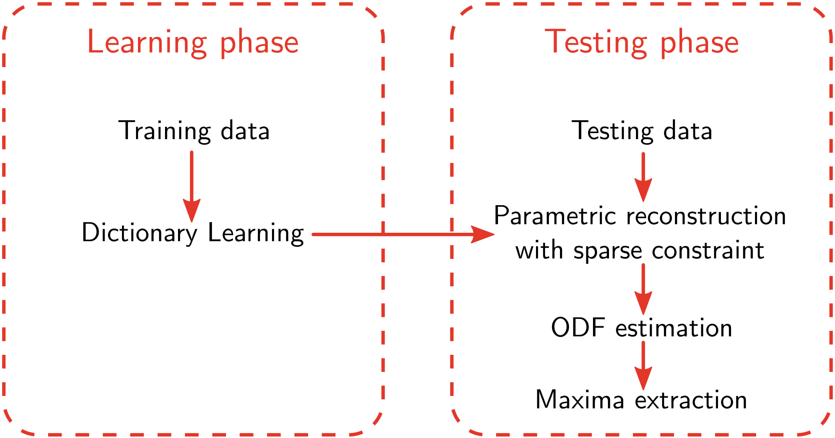
- analytical EAP/ODF reconstruction [J Cheng (MICCAI'10)]
- modified for continuity [E Caruyer (ISBI'12)]

$$C_{n,\ell,m}(q \cdot \mathbf{u}) = \chi_n \frac{q^2}{\zeta} L_n^{5/2}\left(\frac{q^2}{\zeta}\right) e^{-q^2/2\zeta} Y_{\ell,m}(\mathbf{u})$$



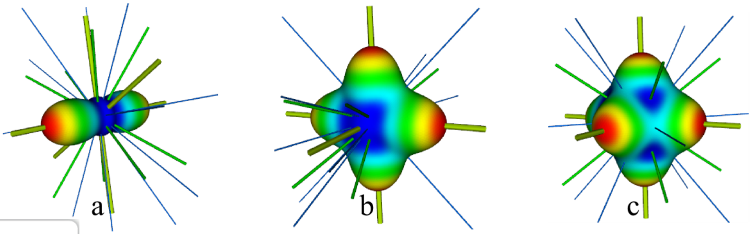


Pipeline


Learning phase
Training data
Dictionary Learning
Testing phase
Testing data
Parametric reconstruction
with sparse constraint
ODF estimation
Maxima extraction

A Polynomial Approach to Maxima Extraction

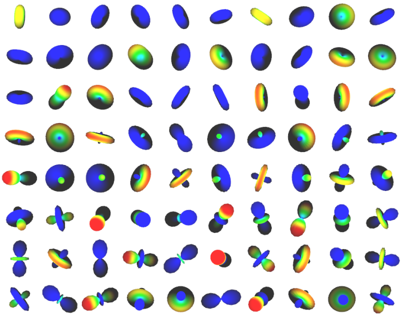
1. Homogeneous Polynomial (HP) representation of ODF
2. Constrained optimization formulation
   - System of homogeneous polynomial
   - Polynomial System solver (bracket ALL roots analytically and refine them numerically with high accuracy)
3. Categorize extrema: maxima, minima, saddle-pointsa
   - Bordered Hessian approach (generalization of Hessian approach to constrained optimization)

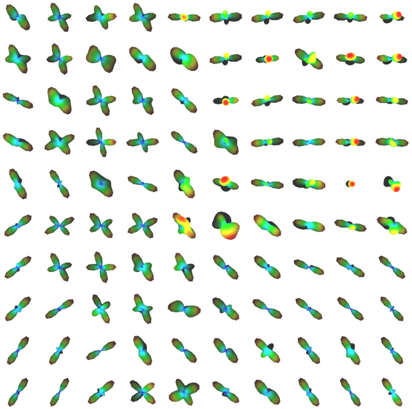


Some results

Training data: 100,000 diffusion models (multi-tensor)

- 1, 2 or 3 tensors (with equal property)
- for each compartment
  - random FA $\in [0.4, 0.7]$, random Trace $\in [0.5, 2.0]\mathrm{mm}^2 \cdot \mathrm{s}$
  - random orientation





Conclusions

- A new sparse representation for diffusion MRI data.
- Analytical EAP/ODF reconstruction.
- Learning and fitting procedure are acquisition-independent.
- Challenging crossing fibers reconstructed with few measurements ($N = 15$)
- Compatible with low SNR.