Fiber Orientation Assessment via Symmetric Tensor Decomposition

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HARDI reconstruction workshop

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From higher-order tensors to fiber orientations

- An ODF may be represented as an even-order homogenous polynomial (symmetric tensor).

$$F(\mathbf{v}) = \sum_{a=0}^{n} \sum_{b=0}^{n-a} c_{ab} v_1^a v_2^b v_3^{n-a-b}, \quad \mathbf{v} \in S^2$$

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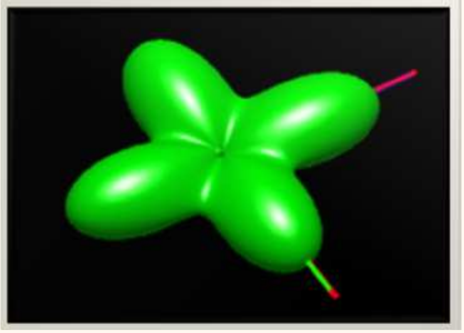
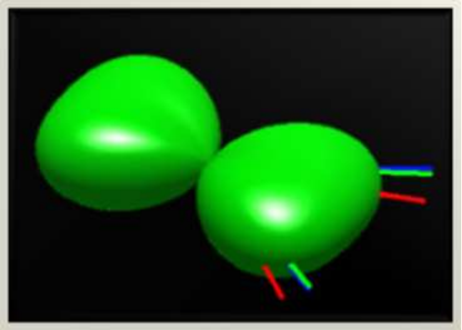
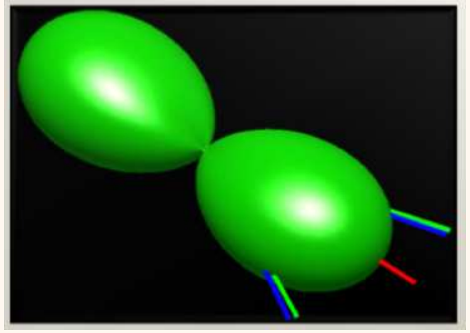
- A low-rank tensor approximation provide accurate estimates of the fiber orientations and enables to reach the full angular resolution of the estimation technique [Schultz et al. IEEE TVCG (2008), Jiao et al., IPMI (2011)]

Higher-order tensor decompositions

$$\mathcal{D} \approx \sum_{r=1}^{k} \lambda_r (\underbrace{\mathbf{v}_r \otimes \mathbf{v}_r \otimes \cdots \otimes \mathbf{v}_r}_{\text{n times}}), \quad ||\mathbf{v}_r|| = 1.$$







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Can we use a symmetric tensor decomposition to estimate the orientations directly from the DWI data ?

Higher-order homogenous polynomials

- Homogenous polynomials may be decomposed as a sum of powers of linear-forms [Common et al., 2010].

$$F(x_1, x_2, \ldots, x_l) = \sum_{i=1}^{r} \lambda_i (\boldsymbol{\alpha}_i \cdot \mathbf{x})^d$$

- We approximate a fODF by powers of linear-forms that represent single fibers

$$F(\mathbf{x}) \approx \sum_{i=1}^{k} (\boldsymbol{\alpha}_i \cdot \mathbf{x})^d, \quad \boldsymbol{\alpha}_i \in \mathbb{R}^3,\ \mathbf{x} \in S^2,\ k < r$$

Spherical Deconvolution

- To estimate the fiber parameters directly from the DWI data we convolve the fODF approximation

$$\min_{\boldsymbol{\alpha}_j} \frac{1}{2} \sum_{i=1}^{n} \left\| S(\mathbf{g}_i, b) - S_0 \int_{S^2} \sum_{j=1}^{k} (\boldsymbol{\alpha}_j \cdot \mathbf{v})^d K(\mathbf{g}_i, \mathbf{v}) d\mathbf{v} \right\|^2$$

- Single fiber response kernel

$$K(\mathbf{g}_i, \mathbf{v}, \delta) = e^{-\delta(\mathbf{g}_i^T \cdot \mathbf{v})^2}$$

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- $\mathbf{u}_j = \frac{\tilde{\boldsymbol{\alpha}}_j}{\|\tilde{\boldsymbol{\alpha}}_j\|}$
- $w_j = \frac{\|\tilde{\boldsymbol{\alpha}}_j\|^d}{\sum_{j=1}^{k} \|\tilde{\boldsymbol{\alpha}}_j\|^d}$

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- Sampling scheme: b=3000, 64 directions

Thank you !