Evaluation of the Deconvolved Diffusion Spectrum Imaging Technique

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- Description of the Reconstruction Method
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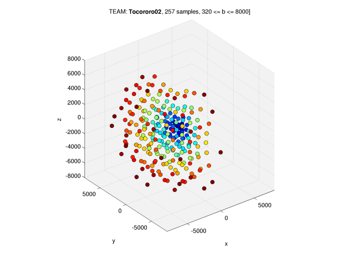
Sampling scheme

Details

- $N = 257$ points in one hemisphere (volume).



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- $N = 257$ points in one hemisphere (volume).
- The signal was duplicated on the other hemisphere to obtain 515 points [Wedeen V.J. et al., 2005].

References

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- $N = 257$ points in one hemisphere (volume).
- The signal was duplicated on the other hemisphere to obtain 515 points [Wedeen V.J. et al., 2005].
- $320 \leq$ b-value $\leq 8000 \quad s/mm^2$
- Two datasets: isolated voxels (IV) and structured field (SF). *SNR* $=$ 5, 10, 15, 20, 25, 30, 35 and 40. (http://hardi.epfl.ch/contest.html)

References

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Description of the Reconstruction Method

Pipeline

1. Conventional DSI Reconstruction
2. Deconvolved DSI
3. *ODF* Computation
4. *ODF* Maxima Extraction

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Conventional DSI Reconstruction

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Diffusion Spectrum Imaging (DSI)

$$P(r_x, r_y, r_z) = \iiint\limits_{V=\{-\infty,\infty\}} E(q_x, q_y, q_z) e^{-2\pi i (r_x q_x + r_y q_y + r_z q_z)} dq_x dq_y dq_z,$$

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⇓ Diffusion Propagator ⇓ Signal

where $[r_x, r_y, r_z]$ and $[q_x, q_y, q_z]$ are the Cartesian real- and q-space vectors.

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where $[r_x, r_y, r_z]$ and $[q_x, q_y, q_z]$ are the Cartesian real- and q-space vectors.

$$ODF(\hat{\mathbf{r}}) = \int_0^\infty P(\rho, \hat{\mathbf{r}}) \rho^2 d\rho, \quad \Rightarrow \text{Orientational Distribution Function}$$

where $[r_x, r_y, r_z] = \vec{r} = \rho \hat{\mathbf{r}}$ (Spherical coordinates).

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Deconvolved DSI

Description of the Reconstruction Method

Theory

In practice: $\tilde{P} \approx FastFourierTransform(E)$

$$\tilde{P}(r_x, r_y, r_z) = \iiint_{V=\{-\infty,\infty\}} \sqcap(q_x, q_y, q_z) E(q_x, q_y, q_z) e^{-2\pi i (r_x q_x + r_y q_y + r_z q_z)} dq_x dq_y dq_z,$$

where, $\sqcap(q_x, q_y, q_z) = \begin{cases} 1 & \text{if } \sqrt{q_x^2 + q_y^2 + q_z^2} \leq q_{max}, \quad \text{(if } q_{max} = 5,515 \text{ points in } 11 \times 11 \times 11 \text{ grid)} \\ 0 & \text{otherwise.} \end{cases}$

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$\mathcal{F}(gk) = \mathcal{F}(g) \otimes \mathcal{F}(k)$, where $\mathcal{F}()$ denotes the Fourier Transform and $\otimes$ is the convolution operator.

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$\mathcal{F}(\sqcap) \Longrightarrow$ Point Spread Function (PSF) of the experiment.

References

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1. This method conserves the constraints on frequency distributions such as normalization and non-negativeness.
2. It is effective when one know the PSF but know little about the additive noise in the image.
3. Implemented in MATLAB ®: deconvlucy

References

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Biggs, D.S., Andrews, M. Acceleration of iterative image restoration algorithms. *Appl. Opt., 36, 17661775 (1997)*

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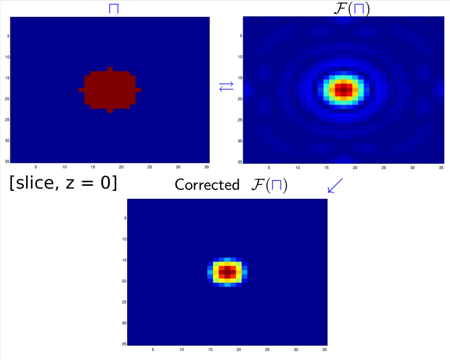
Zero-padding: $35 \times 35 \times 35$ grid



ODF Computation
Description of the Reconstruction Method

Practical aspects:

1. The deconvolved propagator was trilinearly interpolated to a spherical lattice: $P(\rho, \hat{\mathbf{r}}) \equiv P(r_x, r_y, r_z)$
(50 radial points ρ along each of the 724 spatial directions $\hat{\mathbf{r}}$ in the grid).

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2. *ODF* evaluation by taking the radial summation: $ODF(\hat{\mathbf{r}}) = \int P(\rho, \hat{\mathbf{r}})\rho^2 d\rho$

3. *ODF* representation in terms of real spherical harmonics using the regularization approach proposed in [Descoteaux M. et al., 2007].

$$ODF(\hat{\mathbf{r}}) \approx \sum_{l=0}^{L_{max}} \sum_{m=-l}^{l} o_{lm} Y_{lm}(\hat{\mathbf{r}}) \quad (L_{max} = 10 \text{ and } \lambda = 0.004)$$

References

Descoteaux M. et al. Regularized, Fast and Robust Analytical Q-Ball Imaging. *Magn Reson. Med., 58(3), 497-510 (2007)*

ODF Maxima Extraction

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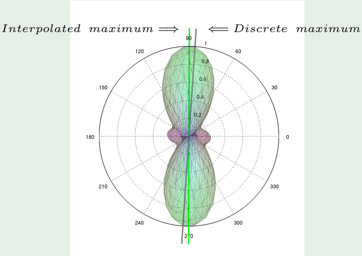
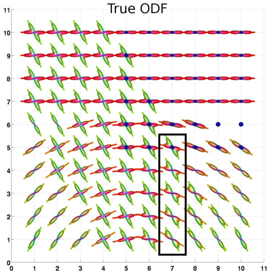
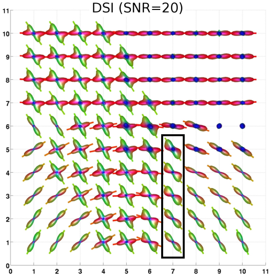
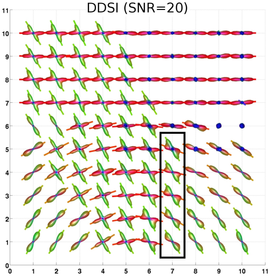


Example: 2D Synthetic Phantom







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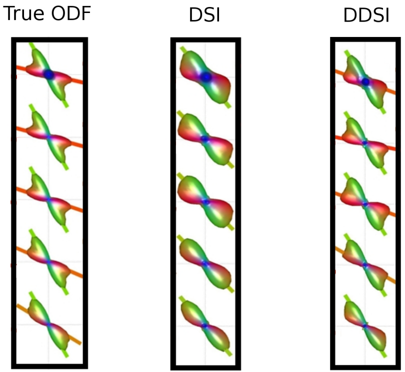
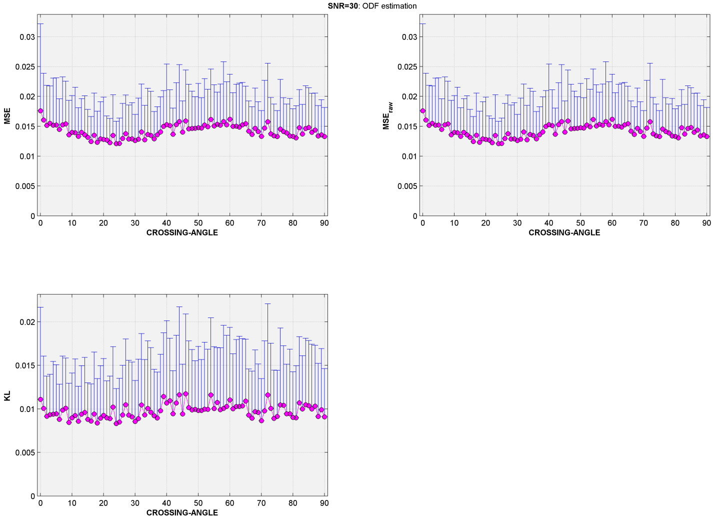
(SNR=20)



Results: Testing IV dataset

ODF estimation based metrics



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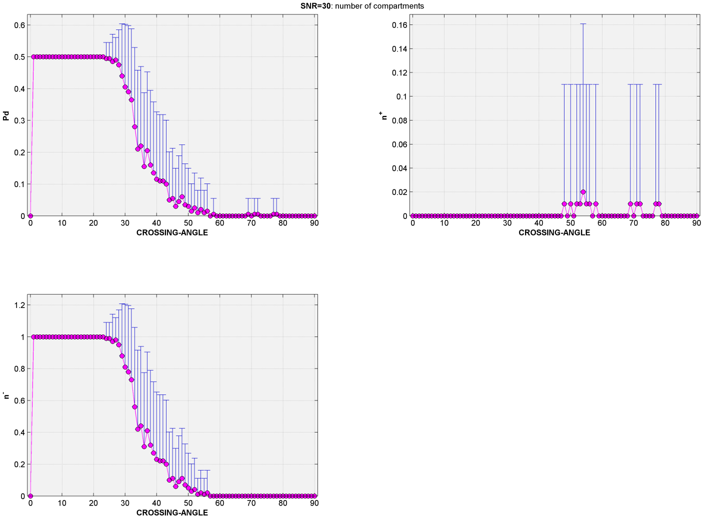

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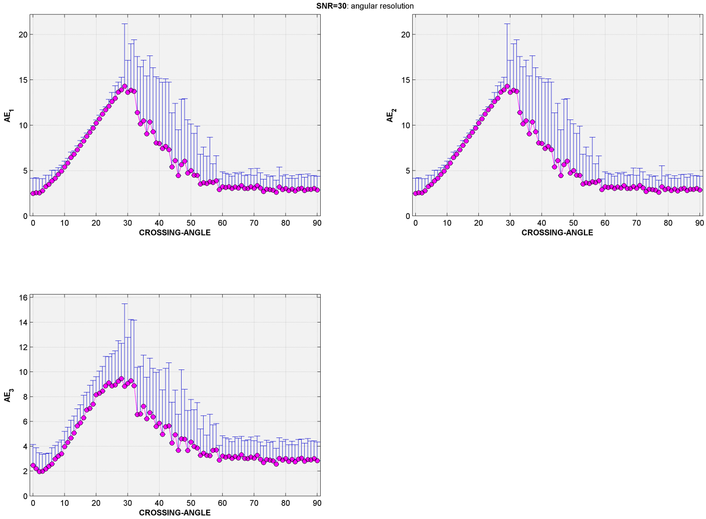
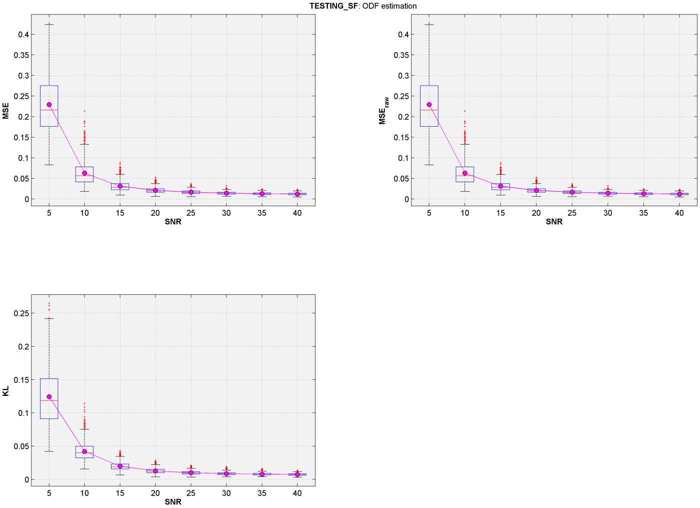


Results: Testing SF dataset
ODF estimation based metrics



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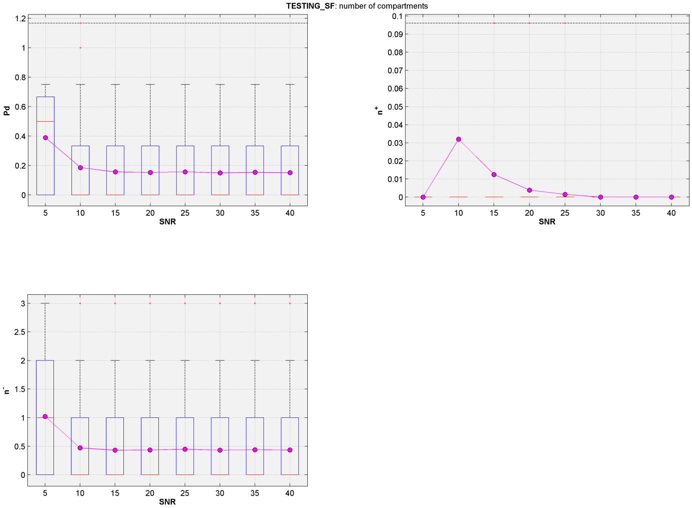

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