High angular resolution diffusion MRI reconstruction through denoising and reweighted ℓ_1−minimization

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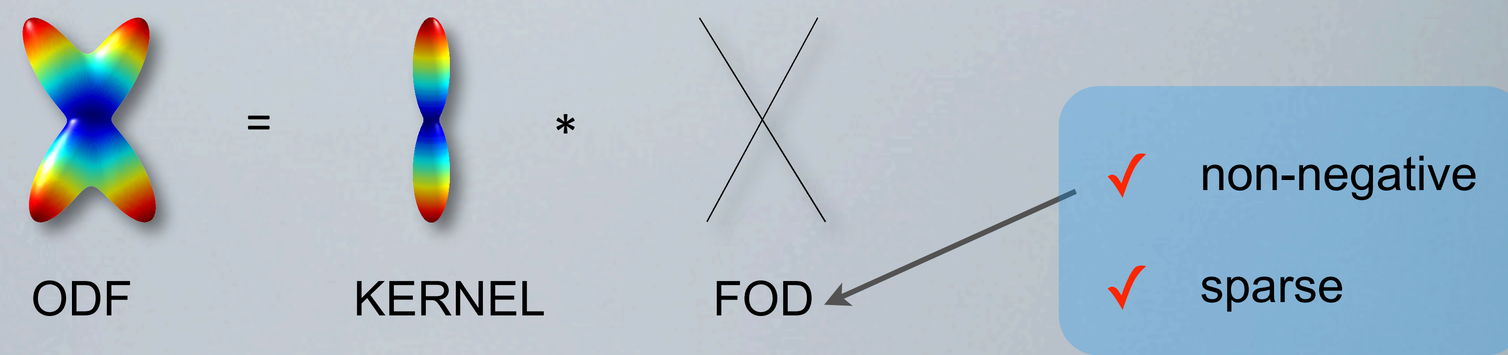
FOD RECOVERY VIA SPHERICAL DECONVOLUTION

Spherical Deconvolution methods assume the Orientation Distribution Function can be expressed as the convolution of a kernel with the Fiber Orientation Distribution:



- ODF: Probability of diffusion along a given direction
- KERNEL:Response generated by a single fiber
- FOD: indicates the **orientation** and the **volume fraction** of the fibers in a voxel

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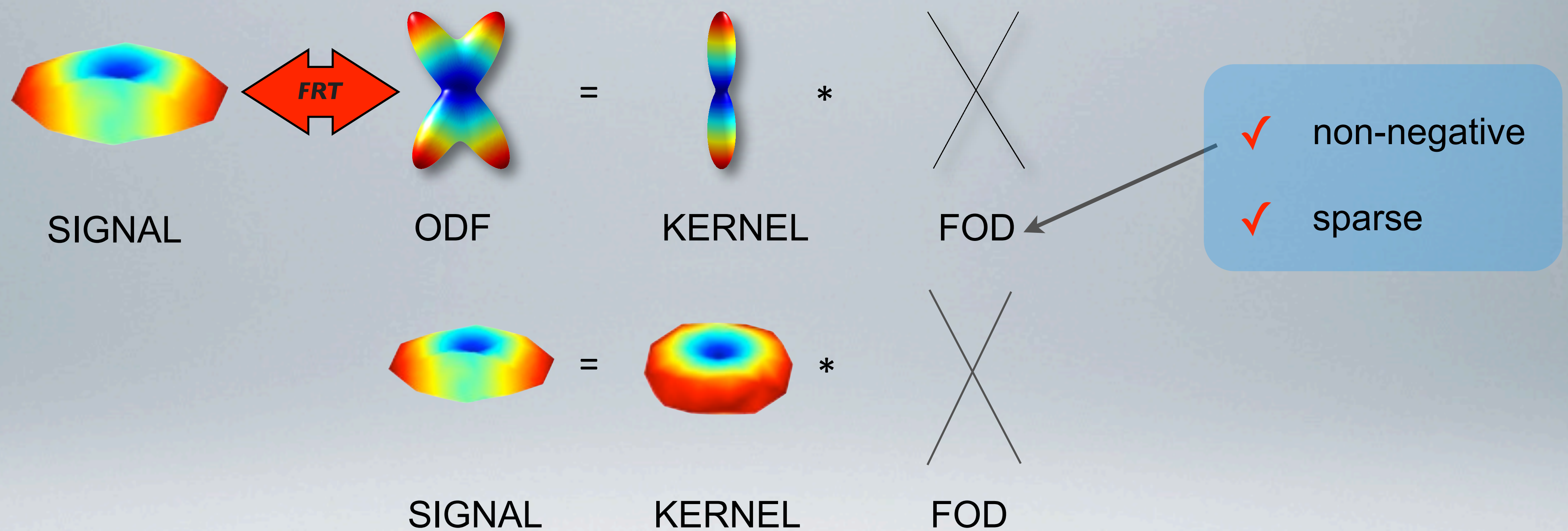

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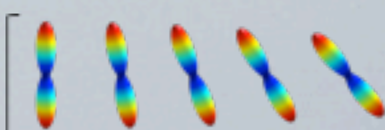
The intra-voxel structure recovery problem can be expressed in terms of the following linear formulation (Jian and Vemuri, 2007):

$$y = \Phi x + \eta$$

- ✓ y is the acquired diffusion MRI data and x is the FOD.
- ✓ Φ is the *sensing basis* or *dictionary* [ ...] ← typically estimated from the data
- ✓ η represents de acquisition noise

State-of-the-Art methods solve the problem for FOD reconstruction using **convex optimization** :

$$\min_{x \geq 0} \underbrace{\|x\|_1}_{\text{sparsity}} \quad \text{s.t.} \quad \underbrace{\|\Phi x - y\|_2 \leq \epsilon}_{\text{data fidelity}}$$

(Ramirez -Manzanares et al., 2007; Jian and Vemuri, 2007)

$$\min_{x \geq 0} \|\Phi x - y\|_2^2 + \underbrace{\beta \|x\|_1}_{\text{sparsity}}$$

(Landman et al., 2012; Pu et al., 2011)

however...

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FOD RECOVERY VIA SPHERICAL DECONVOLUTION

ℓ_1 norm prior is inconsistent!!

$$\min_{x \geq 0} \|x\|_1 \quad \text{s.t.} \quad \|\Phi x - y\|_2 \leq \epsilon$$

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$$\min_{x \geq 0} \quad \|\Phi x - y\|_2^2 + \beta \|x\|_1$$

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Trying to minimize ℓ_1 norm while we have the physical constraint

$$\|x\|_1 = \sum_i |x_i| = 1$$

sum of the volume fractions of the compartments inside a voxel

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Original formulation: ℓ_0-norm as a prior to minimize sparsity

$$\min_{x\geq 0} \quad \|\Phi x - y\|_2^2 \quad \text{s.t.} \quad \|x\|_0 \leq k$$

sparsity term

FOD RECOVERY VIA SPHERICAL DECONVOLUTION

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$$\min_{x\geq 0} \boxed{\|x\|_1} \quad \text{s.t.} \quad \|\Phi x - y\|_2 \leq \epsilon$$

(Ramirez -Manzanares et al., 2007; Jian and Vemuri, 2007)

$$\min_{x\geq 0} \quad \|\Phi x - y\|_2^2 + \boxed{\beta\|x\|_1}$$

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Trying to minimize ℓ_1 norm
while we have the physical constraint $\|x\|_1 = \boxed{\sum_i |x_i|} = 1$

sum of the volume fractions of the compartments inside a voxel

Original formulation: ℓ_0-norm as a prior to minimize sparsity

ℓ_0 problems are **non convex !**

$$\min_{x\geq 0} \quad \|\Phi x - y\|_2^2 \quad \text{s.t.} \quad \boxed{\|x\|_0 \leq k}$$

sparsity term

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FOD RECOVERY VIA SPHERICAL DECONVOLUTION

Reweighted constrained ℓ_1 minimization (*Candes et al., 2008*):

$$\min_{x \geq 0} \quad \|\Phi x - y\|_2^2 \quad \text{s.t.} \quad \underbrace{\|x\|_1^w \leq k}_{\text{sparsity term}}$$

where

$$\|x\|_1^w = \sum_i w_i |x_i|$$

FOD RECOVERY VIA SPHERICAL DECONVOLUTION

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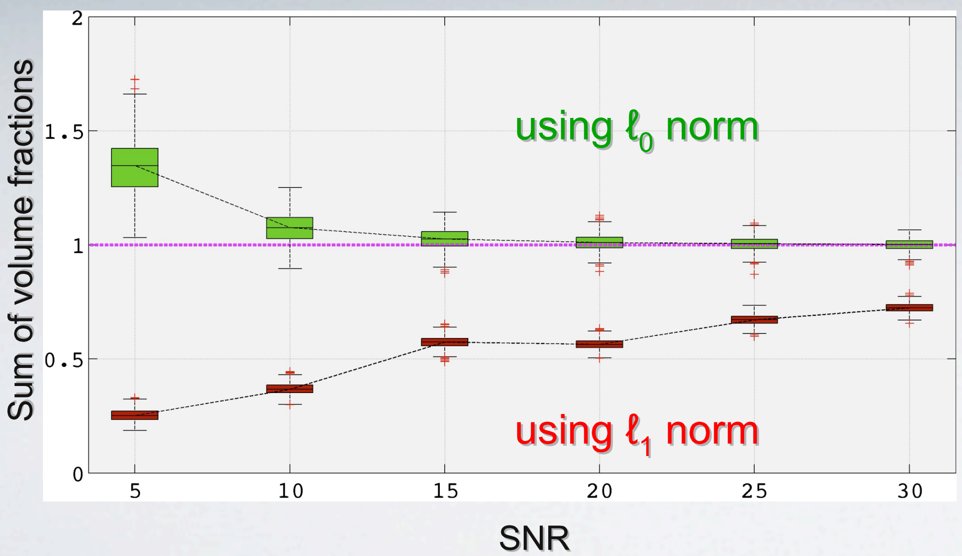
Solving a sequence of these weighted problems with $w_i^{(t)} \approx 1/x_i^{(t-1)}$

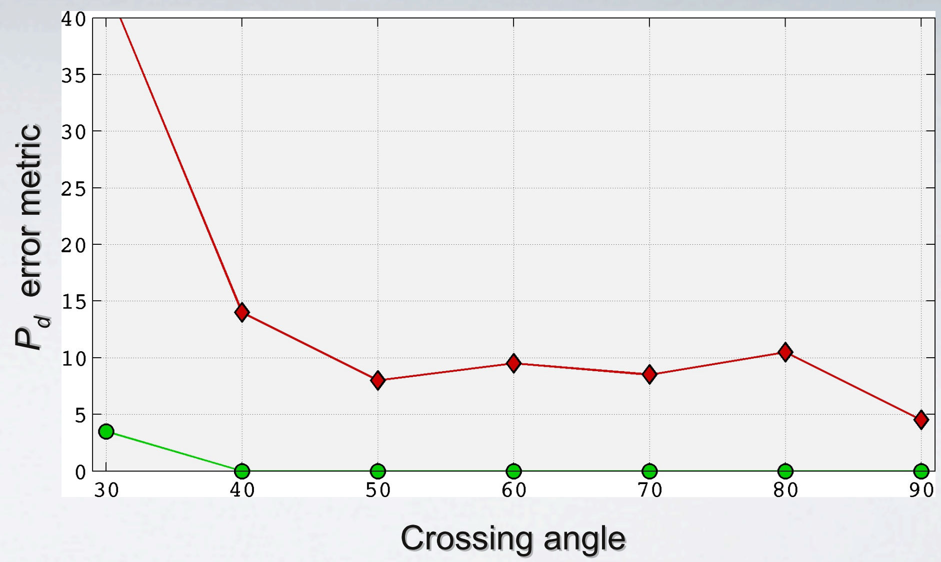
✓ We approximate the ℓ_0 minimization

✓ We solve the ℓ_1 inconsistency

Effect of different regularization priors



Impact on reconstructions



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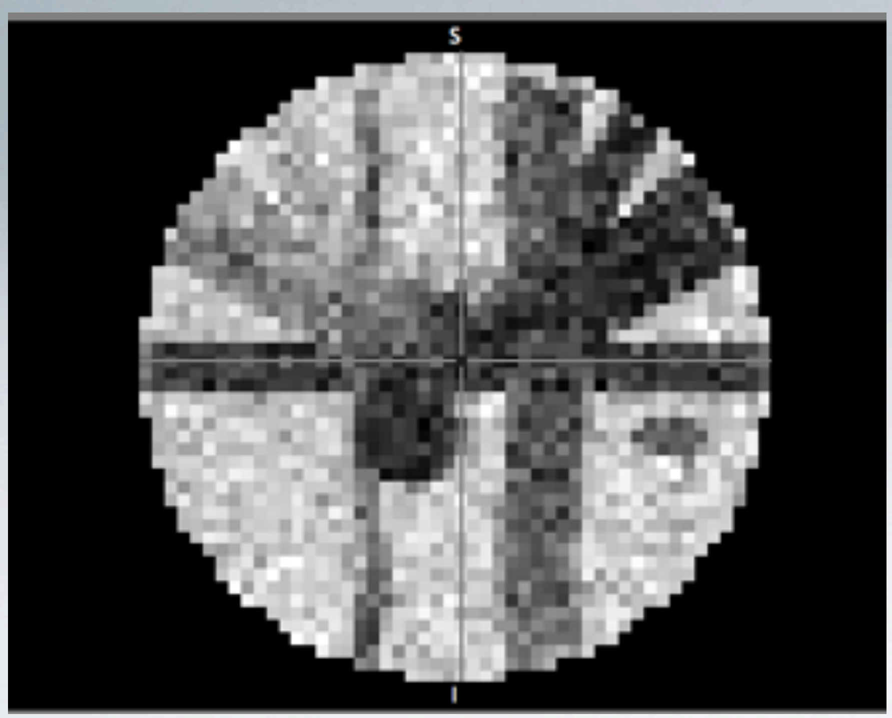
DENOISING STEP

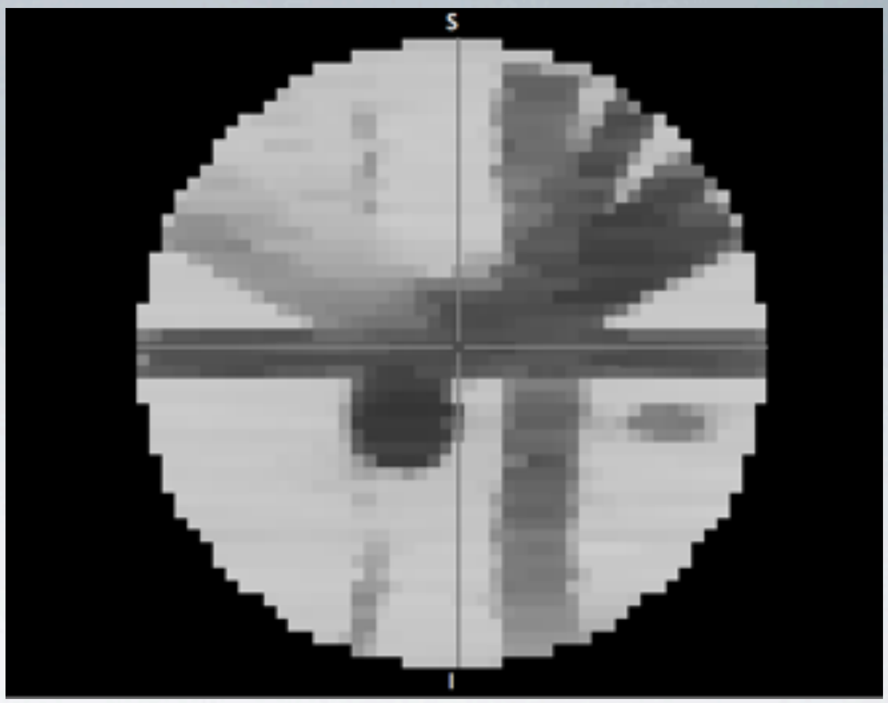
The initial data have been denoised using a ***total variation prior*** for each q-point:

$$\min_{\hat{y}^Z \geq 0} \quad \|\hat{y}^Z\|_{TV} \quad \text{s.t.} \quad \|y^Z - \hat{y}^Z\|_2 < \epsilon, \quad \text{for all } Z.$$

The data is denoised **slice by slice** (2D-TV)**!**



noisy *slice*



denoised *slice*

IMPROVEMENTS: SPATIAL REGULARIZATION

1. Denoise the whole 3D-volume for each q-point at a time (using TV-3D):

$$\min_{\hat{y} \geq 0} \quad \|\hat{y}\|_{TV} \quad \text{s.t.} \quad \|y - \hat{y}\|_2 < \epsilon,$$

where y is a concatenation of all y^Z.

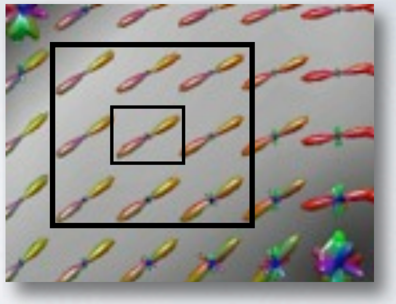
2. Formulate a **global problem** including **spatial regularization**:

$$\min_{\boxed{X \in \mathbb{R}_+^{n \times N^3}}} \quad \|X\|_{0,1} + \boxed{\beta \|X\|_{TV}} \quad \text{s.t.} \quad \|\Phi X - Y\|_2 \leq \epsilon$$

spatial constraint: TV, TGV, ...

solve for all the voxels at the same time,
instead of each voxel sequentially

to exploit spatial coherence in neighboring voxels



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